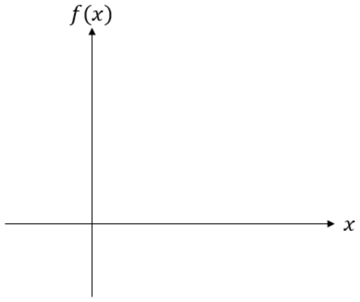
Mathematics Methods

Unit 3

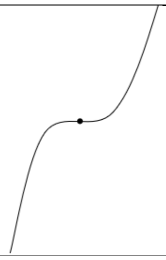
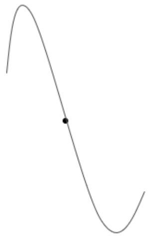
Differentiation – Graphs

1. **Graph (polynomial) of:**

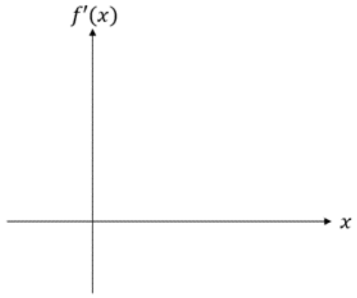
(a) $f(x)$



- $f(x)$ graph that intersects with the horizontal axis (x-axis)/ vertical axis (y –axis) is the x or y intercept(s)
- Stationary point(s) can be seen through the shape of graph as follows:

Concave up (max. point)	
Concave down (min. point)	
Horizontal inflection point	
Oblique inflection point	

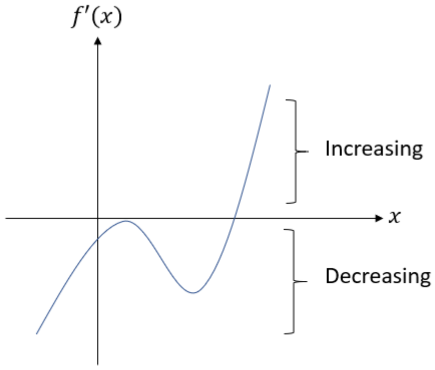
(b) $f'(x)$



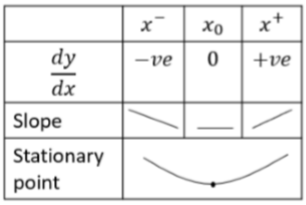
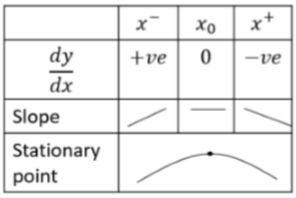
- $f'(x)$ graph that intersects with the horizontal axis (x-axis) may reflect the stationary points (either max., min. or horizontal inflection point) in the graph of $f(x)$.
- *$f'(x)$ graph:

Below x-axis	Graph of $f(x)$ decreasing
Above x-axis	Graph of $f(x)$ increasing

Ex:



- Maximum or minimum point

<table>
<tr><td>Min. point</td><td>
<table>
<tr><td></td><td>x^-</td><td>x_0</td><td>x^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>$-ve$</td><td>0</td><td>$+ve$</td></tr>
<tr><td>Slope</td><td>╲</td><td>—</td><td>╱</td></tr>
<tr><td>Stationary point</td><td colspan="3"></td></tr>
</table>
*Decreasing to increasing $f'(x)$ graph</td></tr>
<tr><td>Max. point</td><td>
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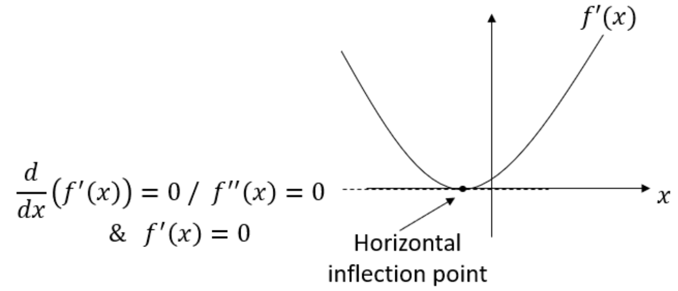
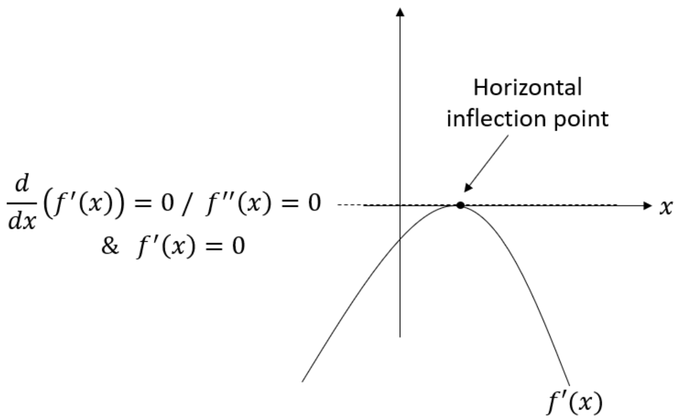
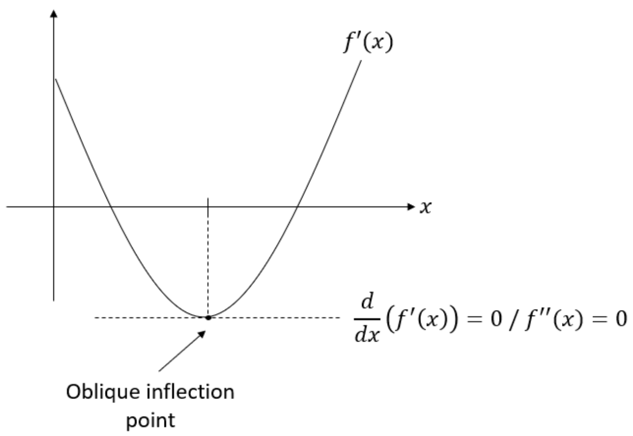
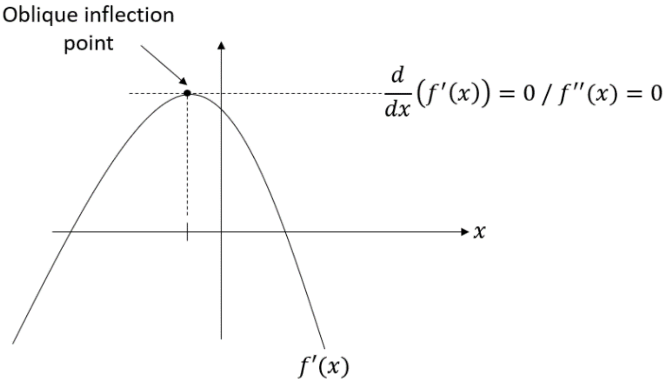
<tr><td>Stationary point</td><td colspan="3"></td></tr>
</table>
*Increasing to decreasing $f'(x)$ graph</td></tr>
</table>

**follow the previous point*

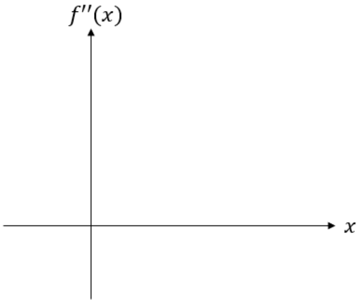
- Inflection point

Horizontal inflection point (HIP)	$f'(x) = 0$ (concave up/ down $f'(x)$ graph touches the x-axis as $f'(x) = 0$) Ex:  *or* 
Oblique inflection point (OIP)	$f'(x) \neq 0$ (concave up/ down $f'(x)$ graph is either above/ below x-axis as $f'(x) \neq 0$) Ex: 

Oblique inflection point (OIP)	*or*  *The maximum/ min point of $f'(x)$ graph is the point of oblique inflection for $f(x)$* graph

- $f'(x)$ values of any point on the graph is the gradient value for graph of $f(x)$

(c) $f''(x)$

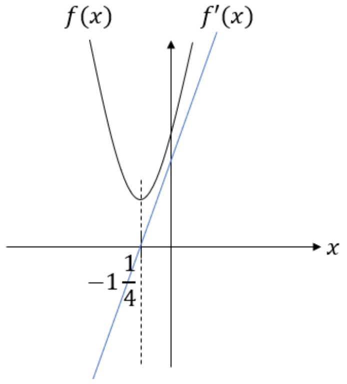


- $f''(x)$ graph that intersects with the horizontal axis (x-axis) reflects the inflection points in graph of $f(x)$.
- If $f''(x)$ graph touches but does not intersect (cross) the horizontal axis (x-axis), concavity does not change, thus it cannot be an inflection point.
- $f''(x)$ graph:

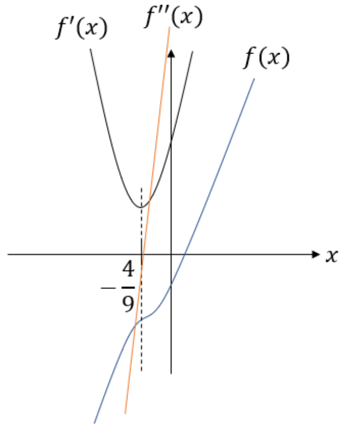
Below x-axis, $f''(x) < 0$	Graph of $f(x)$ concave downwards
Above x-axis, $f''(x) > 0$	Graph of $f(x)$ concave upwards

2. Derivative of various graphs of function

(a) Polynomial

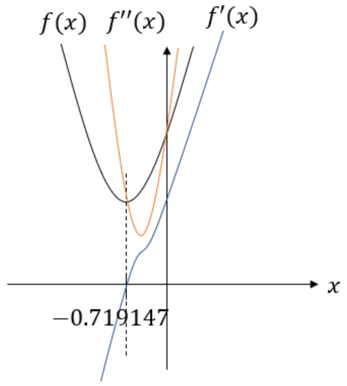
Power of	Example
2	 $f(x) = 2x^2 + 5x + 7$

Analysis	*Reason*
There is an min. point at $x = -1\frac{1}{4}$	$f'(x)$ graph intersects the horizontal axis (x-axis) at $x = -1\frac{1}{4}$ from decreasing to increasing $f(x)$.

Power of	Example
3	 $f(x) = 3x^3 + 4x^2 + 6x - 5$

Analysis	*Reason*
There is an inflection point at $x = -\frac{4}{9}$	$f''(x)$ graph intersects with the horizontal axis (x-axis) at $x = -\frac{4}{9}$. *or* Min. point of $f'(x)$ graph is above the horizontal axis (x-axis).

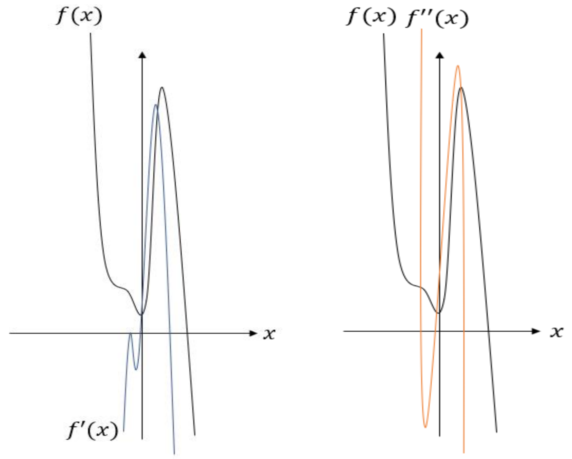
4



$$f(x) = 3x^4 + 3x^3 + 5x^2 + 7x + 17$$

Analysis	*Reason*
There is a min. point at $x = -0.719147$	$f'(x)$ graph intersects the horizontal axis (x-axis) at $x = -0.719147$ from decreasing to increasing $f(x)$.

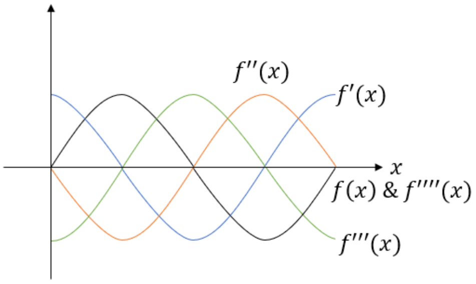
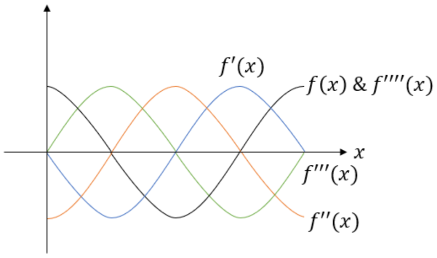
5



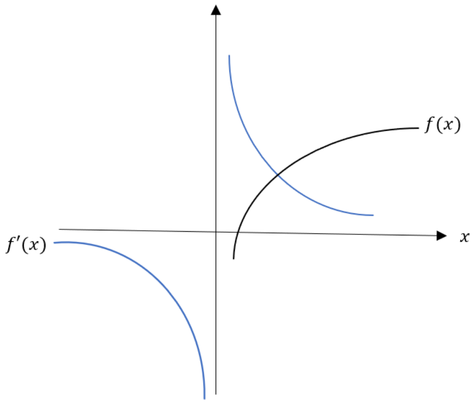
$$f(x) = -4x^5 + 2x^4 + 12x^3 + 7x^2 - x + 5$$

Analysis	*Reason*
There is a min. point at $x = 0.061568$	$f'(x)$ graph intersects the horizontal axis (x-axis) at $x = 0.061568$ from decreasing to increasing $f(x)$.
There is a min. point at $x = 1.69517$	$f'(x)$ graph intersects the horizontal axis (x-axis) at $x = 1.69517$ from increasing to decreasing $f(x)$.
There are inflection points at $x = -0.661023$, $x = -0.2235$ and $x = 1.18452$	$f''(x)$ graph intersects with the horizontal axis (x-axis) at $x = -0.661023$, $x = -0.2235$ and $x = 1.18452$.

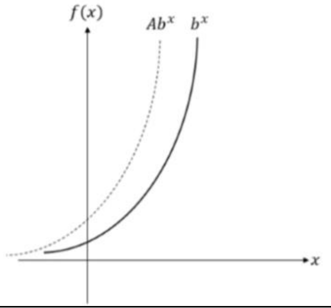
(b) Trigonometric

Graph of	Example
sin	 Derivative graph of sin will be of the shape for graph of sin for every 4th, 8th, 12th,… derivation.
cos	 Derivative graph of cos will be of the shape for graph of cos for every 4th, 8th, 12th,… derivation.

(c) Exponential and logarithmic

Graph	Example
Logarithmic graph	 $f(x) = \log_5 x$

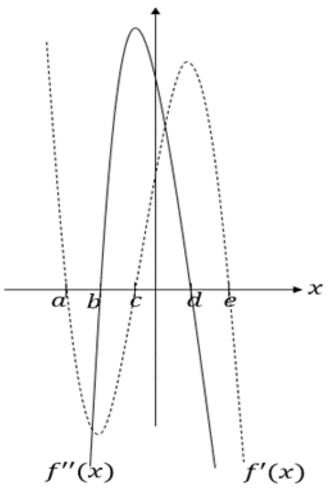
Exponential graph

If function in form of Ae^{x} Ex: 5 e^{x}	Graph $f(x)$ is the same regardless of differentiation.
If function in form of Ae^{bx} Ex: 5 e^{5x}	Graph $f(x)$ is vertically stretched as $\|A\| > 1$. 
Other	Graphs of derivative for graph $f(x)$ varies according to the exponential function, $f(x)$.

3. Exam questions

Example 1:

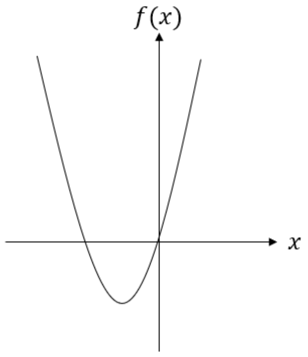
Diagram below shows graphs of $f'(x)$ and $f''(x)$ being plotted in the same graph.



- Identify whether the graph of $f(x)$ has inflection points, maximum point or minimum point. If yes, state the point(s).
- Sketch a graph of $f(x)$.

Example 2:

Diagram below shows a graph of $f(x)$ that has a maximum point at $x = -\frac{1}{3}$.



- What values of x is the gradient of graph positive?
- Sketch a graph of $f(x)$.

END